Steps to Success

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High School Equivalency in the Community

a 'Mail-In' program for adults who did not finish high school

Ninth Step–

NO STOPPING NOW!

LESSON 9
Mathematical Reasoning

ASSIGNMENT 1

A linear equation with one variable usually has one answer. x + 2 = 5, so x = 3. There are no other solutions to this equation.

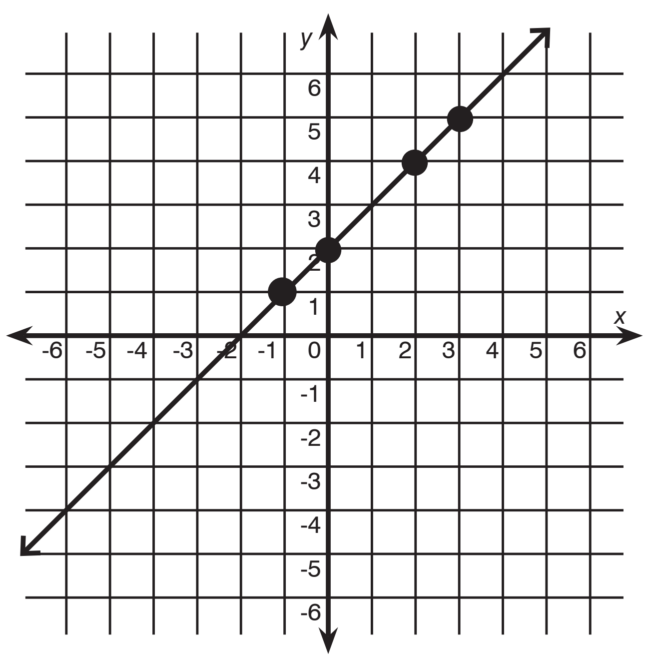
A linear equation with two variables has an infinite number of answers.

x + y = 5 (2,3) is an ordered pair that means x = 2 and y = 3. This is a relation, a function.

How many answers can you think of? …(-2,7), (-1,6), (0,5), (1,4), (6,-1)….and more. Then there are decimal…. (1.5, 3.5), (1.6, 3.4), (1.7, 3.3)… and more.

Rather than write all of these answers out, we can represent **the solution** with a line. Any point in our list is included in the line, and any point on the line will fit into the equation.

Slope is the direction of a line. ***(The slope shows the rate of change in the y variable over the rate of change in the x variable.)*** The line on the graph below goes up one unit for every unit it goes to the right. It is said to have a slope of a positive one.



Slope (m) is defined by the formula:

$$\mathbf{m} = \frac{\text{(the change in } y)}{\text{(the change in } x)} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

Memorize the formula. Slope is m. The small numbers after the x's and y's are called subscripts. Subscripts are used to tell one ordered pair from another x_2 and y_2 come from point two and x_1 and y_1 come from point one. Δy, pronounced "delta y". Δy, is sometimes called "rise" because it describes how much a line changes in the up and down direction. Δx, "delta x", is the change in x. Δx is called "run" because it describes how much a line changes from left to right. The change, or Δ, is found by subtracting the x and y values of one point from another point. ***Slope is the change in y over the change in x, or rise over run***.

EXAMPLE 1

Find the slope of the line containing (7,8) and (3,5).

$$\mathbf{m} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(5-8)}{(3-7)} = \frac{-3}{-4} = \frac{3}{4}$$

Something to remember: If you ever have a zero in your denominator (*a very undesirable situation in the math world*), you have an ***undefined line***. An undefined line is vertical.

EXAMPLE 2

Find the slope of the line containing (2,8) and 2, 5).

$$\mathbf{m} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(5-8)}{(2-2)} = \frac{-3}{0} = \text{undefined}$$

LESSON 9
Mathematical Reasoning

DIRECTIONS
Show your work on a separate sheet of paper.

A. Find the slope of the line containing the given points. Use the slope formula.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

1. **(3,-5) and (-4,7)** ______
2. **(3,-5) and (3,7)** ______
3. **(3,-5) and (-4,-5)** ______
4. **(0,2) and (4,0)** ______
5. **(4,2) and (2,2)** ______
6. **(6,1) and (0,3)** ______

Another way to find the slope is by using ***slope intercept form***. If a linear equation is solved for y, then the coefficient of x is the slope. So, to find the slope of any given equation, just solve for y.

EXAMPLE 3

Find the slope of the line -2x + y = 4

a. Move everything except the y term to the other side of the equation.
y = 2x + 4

b. Divide by the coefficient of y (In this case, the coefficient is 1).

Now you have the ***slope intercept form***:
y = mx + b

Slope m = 2 or $\frac{2}{1} = \frac{\text{rise}}{\text{run}}$

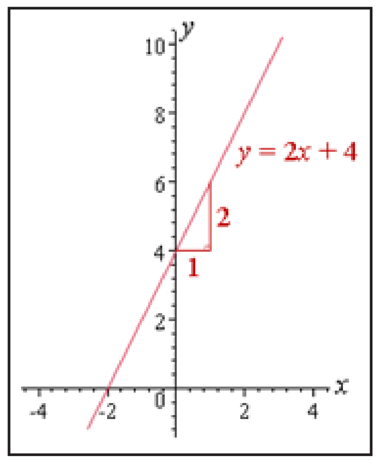
y – intercept b = 4

DIRECTIONS
Show your work on a separate sheet of paper.

B. Find the slope of the y-intercept of the line. Use the y-intercept form: y = mx + b

7. **5x – 3y = 9** ______
8. **2x = 3y + 3** ______
9. **-4x – 7y = -9** ______

Slope intercept form of a line is the most useful form for graphing equation quickly.

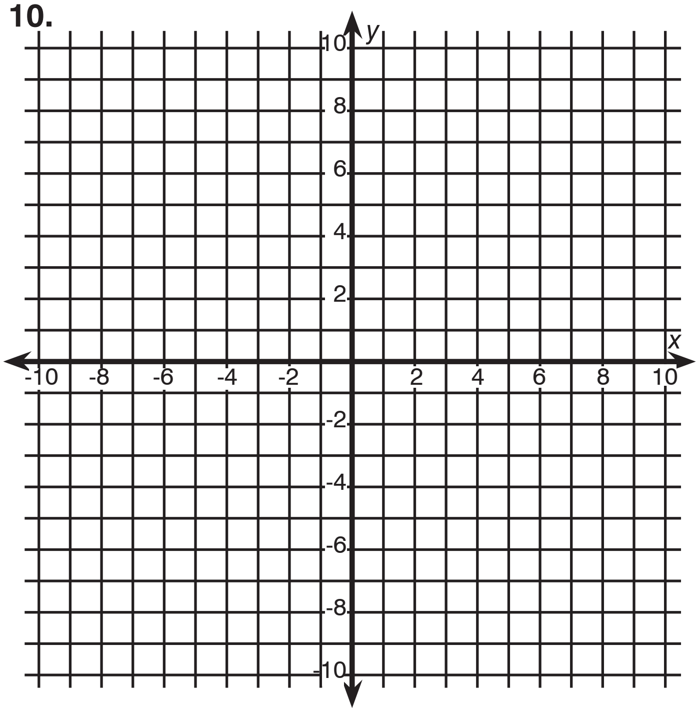


The term b is where the line intersects the y-axis. Since 4 is the b term then the y-intercept is (0, 4). The m coefficient -2- gives the direction of the line. This is all you need to graph a line. Just go to the intercept, then make the slope move over 1 (the run) and up 2 (the rise). This gives you 2 points on your graph. Continue from the last point moving over 1 and up 2 to plot another point. After you have found enough points, usually three, connect them, and you have a graph of the line.

LESSON 9
Mathematical Reasoning

C. Using the graph below, graph the equation: $y = 3x - 4$.

10.



Hint: Slope is $\frac{3}{1}$ and Y intercept is (0,-4)

EXAMPLE 4: **$-3x + y = 2$**

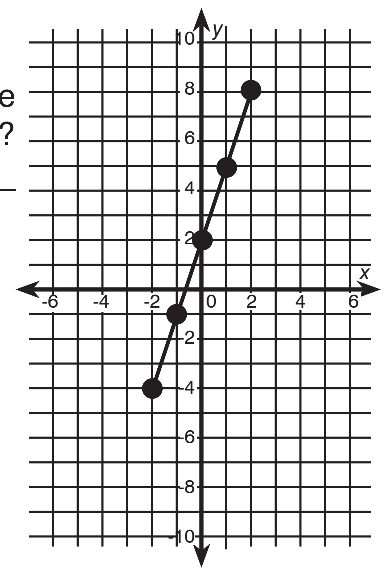
Another way of graphing a line is to just make a chart (a *table of values*) and find ordered pairs that fit into the equation until you have enough points to make a graph.

$-3x + y = 2$

- Choose values for x.
- Substitute the values for x.
- Solve for y

x	y
-2	-4
-1	-1
0	2
1	5
2	8

When you have enough points, connect them for a graph.



Is the slope of this line positive or negative?

____

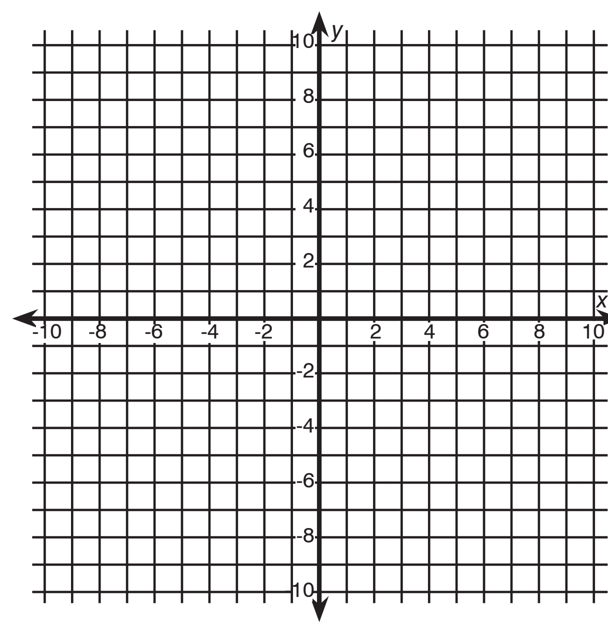
DIRECTIONS

Show your work on a separate sheet of paper.

D. Fill in the *y* column in each table and graph the equation.

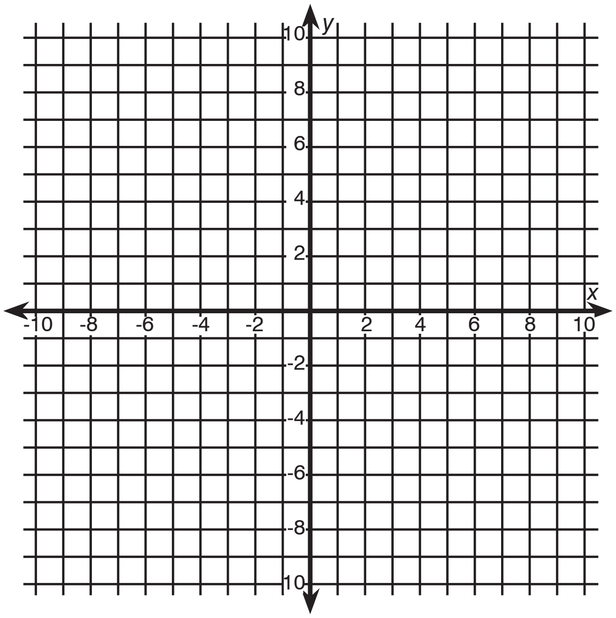
11. $y = \frac{1}{2}x + 3$

x	y
-2	
0	
2	



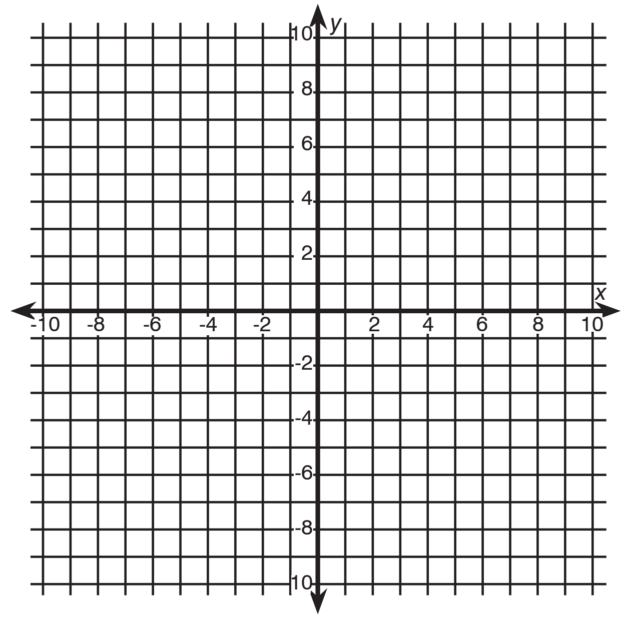
12. **$y + 3x = -1$**

x	y
-1	
0	
1	



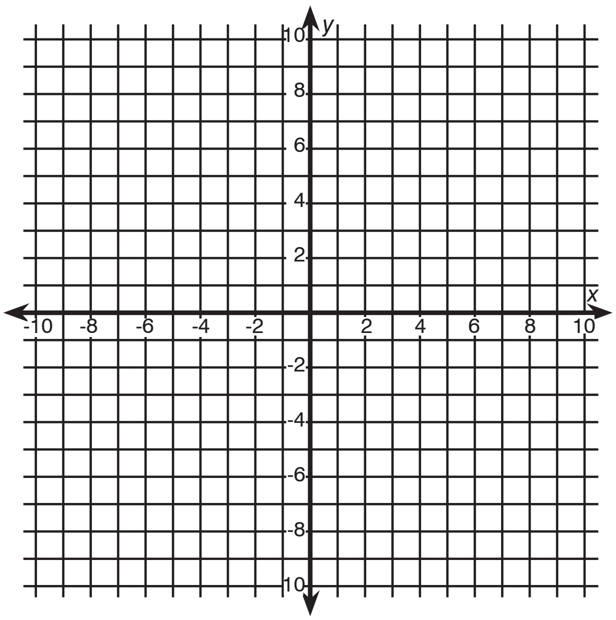
14. **$y = 3 – 2x$**

x	y
0	
1	
2	



13. **$-2 + y = -x$**

x	y
1	
2	
3	



GED TEST TIP

If a linear equation is not written with y on one side of the equation, use inverse operations to isolate y.

Example: **$2x = y = 15$**

Subtract 2x from each side. $y = -2x = 1$

LESSON 9
Mathematical Reasoning

Now put it all together!

ASSIGNMENT 2

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

DIRECTIONS

Use the written information along with the table to complete the following assignment.

- Use the grid to create a coordinate plane.
- Label the x- and y-axis.

1. **This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.**

   The scientist plots the data for tree 2 on a coordinate grid. She begins by plotting data for year 3 and year 11. Graph the locations of the two points on the coordinate grid.

TREE 1

Year	Trunk Diameter (inches)
1	18.6
3	19.2
5	19.8
7	20.4
9	21.0
11	21.6
13	22.2

TREE 2

Year	Trunk Diameter (inches)
1	11.4
3	12.0
5	12.6
7	13.2
9	13.8
11	14.4
13	15.0

2. **This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.**

   The scientist creates an equation that models her data for each tree so that she can predict the diameter in the future. Complete a linear equation that fits the data for tree 1, where *'x'* is the *year* and *'y'* is the *trunk* diameter, in inches.

   Choose from the variables and numbers listed below to complete the equation.

-0.6	18.0	-0.3	18.3	0.3	18.6	0.6	x

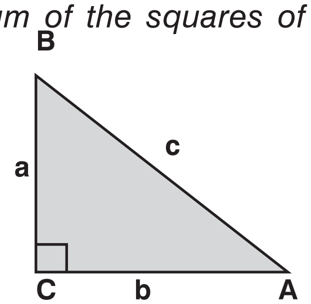
y = _______ _______ + _______

(***HINT:*** *Use the slope formula,* $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ *and the slope intercept form,* $y = mx + b$*.)*

ASSIGNMENT 3

The Pythagorean Theorem is as follows:

In a right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.



The hypotenuse is the side opposite to the right angle (AB) and the legs (BC and CA) are the sides containing the right angle.

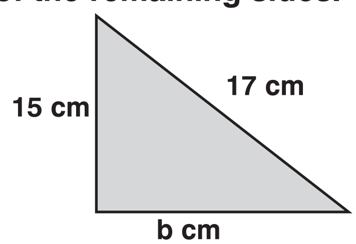
The legs of a right triangle (the two sides of the triangle that meet at the right angle) are customarily labelled as having lengths "a" and "b," and the hypotenuse (the long side of the triangle, opposite the right angle) is labelled as having length "c." Note that the right triangle is denoted by a square in the corner. The lengths are related by the following equation:

$a^2 + b^2 = c^2$ **or** $\sqrt{(a^2+b^2)} = c$

This equation allows you to find the length of a side of a right triangle when they've given you the lengths for the other two sides, and, going in the other direction, allows you to determine if a triangle is a right triangle when they've given you the lengths for all three sides.

EXAMPLE A

Given the right triangles displayed, find the lengths of the remaining sides.



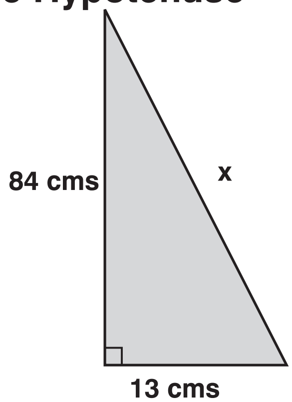
1. Write down the Pythagorean Theorem.
   $a^2 + b^2 = c^2$
2. Substitute in the values you have been given.
   $15^2 + b^2 = 17^2$
3. Using your knowledge of solving equations and the Order of Operations, solve for b.
4.

$$
\begin{aligned}
15^2 + b^2 &= 17^2 \\
225 + b^2 &= 289 \\
b^2 &= 289 - 225 \\
b^2 &= 64 \\
\sqrt{b^2} &= \sqrt{64} \\
b &= 8 \text{ cm}
\end{aligned}
$$

EXAMPLE B

Finding the Length of the Hypotenuse

Find the length 'x.'



'x' is the length of the hypotenuse corresponding to the value 'c' in the formula.

$c^2 = a^2 + b^2$

$c^2 = 84^2 + 13^2$

$c^2 = 7056 + 169$

$\sqrt{c^2} = \sqrt{7225}$

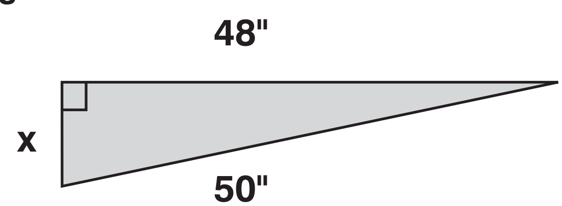
$c = \sqrt{7225} = 85$

The length of the hypotenuse = 85 cms

EXAMPLE C

Finding the Length of a Leg

Find the length of the unknown leg in the diagram below:



We can take a = x, b = 48 and c = 50.

$a^2 + b^2 = c^2$

$a^2 = c^2 - b^2$

$x^2 = 50^2 - 48^2$

$x^2 = 2500 - 2304 = 196$

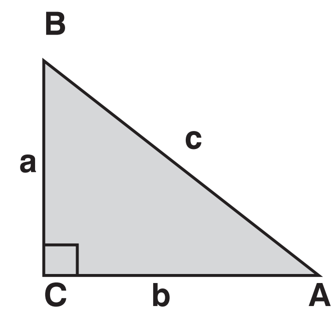
$x = \sqrt{196} = 14$

The length of the leg = 14".

You can also determine if a triangle is a right triangle using the Pythagorean Theorem:

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

EXAMPLE D



If in triangle ABC, $c^2 = a^2 + b^2$ where AB=c, BC=a and CA=b, then triangle ABC is a right triangle.

Let's see if the above ABC triangle is a right triangle. If side a=6, side b=8, and side c (the hypotenuse)=10, you can determine if it is a right triangle by substituting the numbers into the formula: $\mathbf{a^2 + b^2 = c^2}$.

$a^2 + b^2 = c^2$

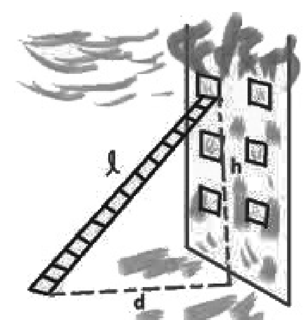
$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$

Triangle ABC is a Right Triangle!

The **Pythagorean Theorem** is used to find the lengths, distances and heights using right triangles which model real life situations.



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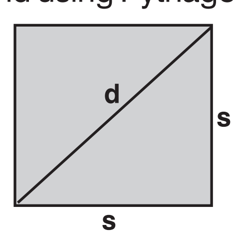
When fire occurs in high raise buildings, the fire fighting men cannot use the regular stairs or lifts. They can reach some floors using ladders. In order to determine the ladder length, they apply the Pythagorean Theorem as they can estimate the height of the floor affected and the horizontal distance they can use to keep the ladder in position.

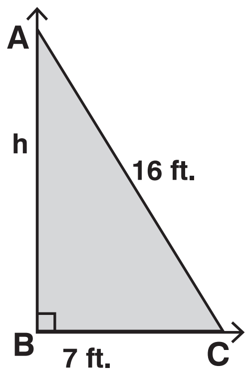
If you own a square plot and propose to construct a diagonal path along the path, the distance of the path can be found using Pythagorean Theorem as the diagonal separates the square or a rectangle into two congruent right triangle and forms the hypotenuse for each of the triangle.



LESSON 9
Mathematical Reasoning

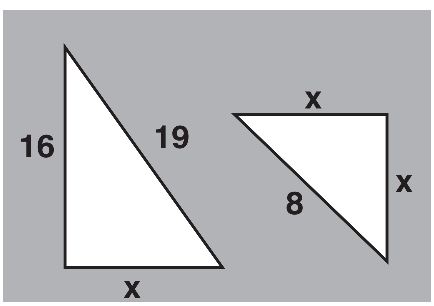
DIRECTIONS
Answer the following questions. Show your work on a separate sheet of paper.

1. **A ladder of length 16 ft. is placed against a wall. If the foot of the ladder is 7 ft. from the wall find the height the ladder reaches on the wall nearest to the tenth of a foot.**



Answer: ______________

2. **Find the value of x in the following triangles rounded to the tenth.**



Answer: ________________

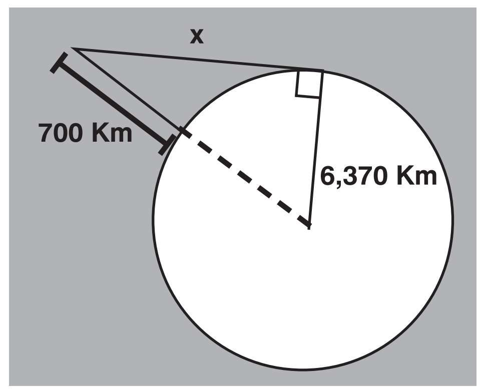
3. **A ramp is placed from a ditch to a main road which is 2 ft. above the ditch. If the length of the ramp is 12 ft., how far away is the bottom of the ramp from the road?**

Answer: ________________

4. **Using the Pythagorean Theorem check whether the numbers 60,63 and 87 can be the measures of the sides of a right triangle.**

Answer: ________________

6. **A Satellite is orbiting the Earth at a height of 700 Kms. Assuming the radius of the earth to be 6370 Kms, find the distance of the Earth's Horizon from the Satellite nearest to a Km.**



Answer: ________________

LESSON 9
Mathematical Reasoning

ASSIGNMENT 4
Distance Formula

The Pythagorean distance formula between the two points is derived using the Pythagorean Theorem. To find the distance between the two points (x_1,y_1) and (x_2,y_2) of ordered pairs use the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE

Find the distance between the points (-5,5) and (2,-9)

Solution: Use the given formula for finding the midpoint of a line.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between x-coordinates:

$\Delta x = 2 - (-5) = 7$

Distance between y-coordinates:

$\Delta y = -9 - 5 = -14$

$d = \sqrt{(7^2 + 14^2)}$

$= \sqrt{(49^2 + 196^2)}$

$= \sqrt{245}$

$= 15.65$

DIRECTIONS
Solve the following math problems. Show your work on a separate sheet of paper.

1. **Find the distance between the points (5,-8) and (3,4).**

   Answer: _______________

2. **What is the slope of the segment connecting the points (2,-9) and (5,3). Find the slope distance between the given points.**

   Answer: _______________

References

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