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High School Equivalency in the COMMUNITY

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Sixth Step– SENSATIONAL WORK!

LESSON 6
Mathematical Reasoning

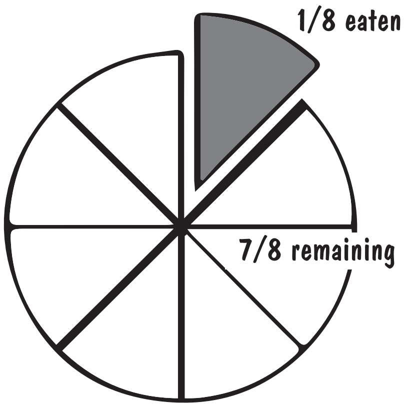
ASSIGNMENT 1

Your fraction book should be complete before beginning this lesson.

Fractions, like decimals and percents, represent parts of something. Fractions describe parts of things like hours, cups, and inches.

The ***denominator*** (bottom number) of a fraction shows how many parts are in a group or how many pieces a whole object is divided into. The ***numerator*** (top number) tells how many of those pieces we're interested in. A ***mixed number*** contains wholes and fractional parts ($23\frac{1}{2}$, for example).

If a pizza is cut into 8 slices, it is cut into eighths. If Jerome eats one piece, he has eaten one eighth of the pizza. Seven eighths of the pizza remains.



When adding decimal numbers, we line up the decimal points so that we add the same kinds of parts together (tens, ones, tenths, hundredths, etc.). The "parts" must match when adding or subtracting fractions, too. In other words, the denominators must be the same.

To find the least common denominator:

If the largest denominator in the problem is evenly divisible by the other denominator(s), it is the common denominator. Evenly divisible means there is no remainder after dividing.

Example: 12 is evenly divisible by 2, 3, and 4 but not 5.

If the largest denominator is not evenly divisible by the other denominator(s), try the largest denominator times 2, then 3, and so forth until you find a number that's divisible evenly by the other denominator(s).

Example: If you are adding or subtracting with $\frac{3}{4}$ and $\frac{1}{2}$, 4 is the least common denominator because 4 is evenly divisible by 2. If you are working with 1/6 and 1/8, 8 will not work because $8 \div 6$ leaves a remainder. 8×2 is 16, but $16 \div 6$ leaves a remainder. 8×3 is 24, and 24 is divisible by 6, so the least common denominator for 1/6 and 1/8 is 24.

To raise a fraction to higher terms:

1. Write a new fraction with the common denominator beside the original fraction.
2. Ask, "What is multiplied by the old denominator to get the new denominator?" Use that number, times the old numerator, to find the new numerator. Put another way, multiply both the numerator and denominator by the same number to find a fraction of the same value using the new (common) denominator.

Example:

$$\frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$$
$$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$$

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To add or subtract fractions:

1. If the denominators don't match, find a common denominator. Raise fractions that don't have it already to higher terms.
2. Add or subtract the numerators; keep the same denominator. You may know from experience that ½ + ¼ = ¾. Let's see how this works with the process given above.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

I multiplied both the numerator and denominator by 2. Now we have the fraction $^2/_4$.

Since we now have the same denominator for both fractions, we can add them. Keep the denominator of 4 and add the numerators (2+1).

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

To reduce your answer:

A fraction is in ***lowest terms*** when there is no number that divides evenly into both the numerator and the denominator.

1. Reducing is the opposite of raising to higher terms. Can you divide the numerator and denominator by 2? 3? 5? 7? etc.
2. Divide both the numerator and the denominator by that same, common number. Check your result—is there yet another number that divides evenly into both?

$$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4} \qquad \frac{9}{12} \div \frac{3}{3} = \frac{3}{4}$$

$$\frac{12}{24} \div \frac{12}{12} = \frac{1}{2} \qquad \frac{35}{75} \div \frac{5}{5} = \frac{7}{15}$$

If the numerator of the answer is larger than its denominator (an improper fraction), ***simplify***:

1. Subtract the denominator from the numerator. Use the result as the new numerator.
2. Place a 1, representing the group subtracted, to the left of the new fraction. For example, to simplify $^{15}/_8$, take 15 - 8 = 7; place a 1 to the left of the fraction. The simpler way of saying $^{15}/_8$ is $1^7/_8$.

Sometimes ***borrowing*** is necessary when the numerator of the fraction being subtracted is larger than the numerator it's being subtracted from. Borrow from the one's digit of a mixed number. This borrowed one comes across as a group the same as the denominator being used, which is added to the numerator of the too-small fraction.

A shortcut is to add the numerator and denominator of the too-small fraction to make the new numerator.

$\overset{3}{\cancel{4}}\,\frac{\overset{11}{\cancel{3}}}{8}$ $-\,2\frac{5}{8}$ $1\frac{6}{8} = 1\frac{3\times2}{4\times2} = 1\frac{3}{4}$	Since $\frac{5}{8}$ is bigger than $\frac{3}{8}$, we had to "borrow" from the **4**. To do this, we took **1**, or $\frac{8}{8}$ from the **4**, changed it into a **3**, and changed $\frac{3}{8}$ to $\frac{8+3}{8} = \frac{11}{8}$.

LESSON 6
Mathematical Reasoning

ASSIGNMENT 1A

DIRECTIONS

Solve the following problems. Show your work on a separate sheet of paper.

1. $\frac{2}{9} + \frac{2}{9}$
2. $\frac{2}{7} + \frac{3}{7}$
3. $\frac{1}{8} + \frac{3}{8}$
4. $\frac{3}{4} + \frac{1}{4}$
5. $\frac{9}{10} + \frac{3}{10}$
6. $\frac{7}{8} + \frac{7}{8}$
7. $\frac{2}{3} + \frac{1}{6}$
8. $\frac{1}{16} + \frac{7}{8}$
9. $\frac{1}{6} + \frac{3}{4}$
10. $\frac{2}{3} + \frac{1}{8}$
11. $\frac{1}{2} + \frac{1}{10} + \frac{1}{5}$
12. $\frac{1}{4} + \frac{3}{10} + \frac{1}{2}$
13. $\frac{1}{6} + \frac{1}{4} + \frac{5}{8}$
14. $12\frac{1}{3} + 19\frac{1}{3}$
15. $3\frac{1}{2} + 6\frac{1}{2}$
16. $12\frac{2}{3} + 14\frac{2}{3}$
17. $7\frac{5}{8} + 3\frac{1}{16}$
18. $2\frac{2}{3} + 4\frac{4}{5} + 6\frac{7}{10}$
19. $\frac{10}{11} - \frac{3}{11}$
20. $\frac{4}{9} - \frac{2}{9}$
21. $\frac{3}{4} - \frac{1}{4}$
22. $\frac{7}{8} - \frac{3}{8}$
23. $\frac{1}{2} - \frac{1}{4}$
24. $\frac{1}{2} - \frac{1}{8}$
25. $\frac{2}{3} - \frac{1}{6}$
26. $\frac{7}{10} - \frac{1}{5}$
27. $7\frac{7}{8} - 4\frac{3}{8}$
28. $14\frac{3}{10} - 13\frac{1}{10}$
29. $5\frac{3}{4} - 2\frac{3}{8}$
30. $7\frac{3}{10} - 2\frac{1}{5}$
31. $27\frac{2}{5} - 14\frac{4}{5}$
32. $4\frac{1}{4} - 3\frac{3}{4}$
33. $3 - 1\frac{1}{2}$
34. $11\frac{3}{8} - 6\frac{3}{4}$
35. $12\frac{7}{12} - 4\frac{7}{8}$
36. $8\frac{2}{3} - 6\frac{7}{8}$

LESSON 6
Mathematical Reasoning

Multiplying and dividing fractions:

When multiplying numbers larger than 1, the answer is larger than the original numbers. However, multiplying decimal or fraction numbers smaller than 1 always results in a smaller product. It helps to think of multiplication with fractions using the word *of*. For example, half *of* a quarter candy bar is a smaller piece than a half or a quarter of it.

To ***multiply*** fractions, multiply straight across the top numbers and straight across the bottom numbers. Reduce your result.

Example:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$$

To avoid reducing, you may ***cancel*** (divide out, as in reducing) any common factors from the numerators and the denominators.

Example:

$$\frac{3}{10} \times \frac{5}{6} = \frac{\cancel{3}^{1} \times \cancel{5}^{1}}{\cancel{10}_{2} \times \cancel{6}_{2}} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

When multiplying whole numbers with fractions, place the whole number over a denominator of 1 so that it looks like a fraction.

To multiply with a mixed number, change it to an ***improper fraction***:

1. Multiply the denominator of the fraction by the whole number, then add the numerator. This is your new numerator.
2. Keep the same denominator.

Example:

$$3\frac{1}{4} \div \frac{3}{16} = \frac{(4\times3)+1}{4} \div \frac{3}{16} = \frac{13}{4} \div \frac{3}{16}$$

When dividing by numbers larger than 1, the answer is smaller than the original numbers. However, dividing by a number smaller than 1 always gives a larger result.

To ***divide*** with fractions:

1. Change any whole or mixed numbers to improper fractions.
2. Keep the first number the same, and change the division sign to multiplication.
3. Invert (turn upside-down) the second number in the problem.
4. Multiply using the rules for multiplication.

Example: $3\frac{1}{4} \div \frac{3}{16}$

$$\frac{13}{4} \div \frac{3}{16} = \frac{13}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{3} = \frac{13}{1} \times \frac{4}{3} = \frac{13\times4}{1\times3} = \frac{52}{3} = 17\frac{1}{3}$$

ASSIGNMENT 1B

DIRECTIONS

Solve the following problems. Show your work on a separate sheet of paper.

1. $\frac{3}{7} \times \frac{2}{5}$
2. $\frac{3}{4} \times \frac{8}{9}$
3. $\frac{1}{8} \times 3$
4. $4\frac{2}{3} \times 5\frac{1}{8}$
5. $\frac{2}{3} \times \frac{4}{9} \times \frac{3}{8}$
6. $1\frac{1}{4} \times 2\frac{3}{10} \times 8\frac{1}{2}$
7. $\frac{1}{4} \div \frac{7}{9}$
8. $\frac{1}{2} \div \frac{1}{4}$
9. $\frac{1}{2} \div \frac{3}{8}$
10. $3 \div \frac{1}{6}$
11. $12\frac{1}{12} \div 4\frac{1}{8}$
12. $\frac{7}{8} \div 4\frac{3}{8}$

LESSON 6
Mathematical Reasoning

Ratios and Proportions

- A ***ratio*** is a comparison of two quantities.

  Examples of ratios:

  2 wins to 3 losses, 1 teacher to 6 students, 25 miles to 1 gallon of gas
- Ratios can be written 3 ways: 2 wins to 3 losses, 2:3, or 2/3
- Always reduce rations to lowest terms. Just remember that you are comparing two number, so ratios can be written as fractions even if the denominator is 1.

Example: A class room has 8 girls and 10 boys. What is the ration of girls to boys?

$$\frac{\text{girls}}{\text{boys}} \quad \frac{8}{10} = \frac{4}{5} \qquad \text{girls:boys } 4:5$$

ASSIGNMENT 2

DIRECTIONS

Write each ratio as a fraction. Be sure to reduce to lowest terms.

1. 18 girls to 20 boys ____________________

2. 15 apples to 25 oranges ________________

3. 12 wins to 6 losses ___________________

4. The Lions played a total of 18 volleyball games. They won 15 and lost 3.

a. What is the ratio of games played to losses? ____________________

b. What is the ratio of games won to games lost? ____________________

c. What is the ratio of games lost to total games played? ____________________

d. What is the ratio of games lost to games won? ____________________

ASSIGNMENT 3

A ***proportion*** sets two ratios equal to one another.

Example:

$$\frac{2}{3} = \frac{6}{9}$$

How do you know this is a proportion? That means 2 ratios of equal value. You can tell if 2 ratios are proportions by cross multiplying. If you get the same product, it is a proportion.

$$\frac{2}{3} = \frac{6}{9}$$
$$2 \times 9 = 18$$
$$3 \times 6 = 18$$
$$18 = 18$$

Ratios and Proportions are very useful in solving many different kinds of word problems. Let's look at one.

Example: If Marcia used 5 gallons of gas to travel 110 miles, how many gallons of gas will she need to travel 209 miles? Proportions are particularly helpful when comparing two items.

- First write down what is being compared. In this case, gallons and miles are being compared.

$$\frac{\text{Gallons Used}}{\text{Gallons Needed}} = \frac{\text{Miles Traveled}}{\text{Miles to Travel}}$$

LESSON 6
Mathematical Reasoning

Set up the ratios carefully, being sure the same type of item is in the same position in each ratio. Note the gallons and miles in the above example.

- Then substitute the numbers for words.

$$\frac{5}{110} = \frac{g}{209}$$

Think of it like this. If Marcia uses 5 gallons of gas for 110 miles, how many gallons of gas will she need for 209 miles? To find the answer to that question:

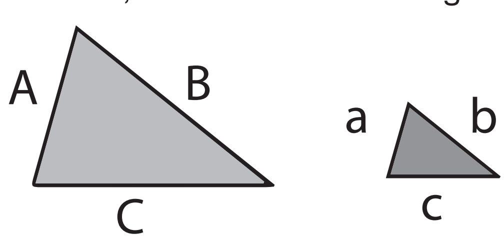
- Multiply the two numbers that are across from each other. The numbers will not always be in the same positions.
- 5 x 209 = 1045
- Divide the product (1045) by the number that is across from the variable (the unknown).
- 1045 ÷ 110 = 9.5 gallons

Proportions are also used to solve problems involving scale, similar triangles, and percents.

Let's look at proportion problems with "similar figures". "*Similar*" is a geometric term, referring to geometric shapes that are the same, except that one is larger than the other. Think of what happens when you use the "enlarge" or "reduce" setting on a copier, or when you get an eight-by-ten enlargement of a picture you really like, and you'll have the right idea.

In the context of ratios and proportions, the point is that the corresponding sides of similar figures are proportional.

For instance, look at the similar triangles below:



The "corresponding sides" are the pairs of sides that "match", except for the enlargement/reduction aspect of their relative sizes. So **A** corresponds to **a**, **B** corresponds to **b**, and **C** corresponds to **c**.

Since these triangles are similar, the pairs of corresponding sides are proportional. That is,

A : a = B : b = C : c.

This proportionality of corresponding sides can be used to find the length of a side of a figure.

Example:

In the displayed triangles, the lengths of the sides are given by:

A = 48 mm
B = 81 mm
C = 68 mm
a = 21 mm

Find the lengths of sides b and c, *rounded* to the nearest whole number.

I'll set up my proportions, using ratios in the form (big triangle length) / (little triangle length), and then I'll solve the proportions. Since I have the length of only side **a** for the little triangle, my reference ratio will be **A : a**.

First, I'll find the length of **b**.

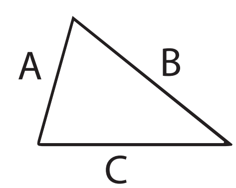
$$\frac{A}{a} = \frac{B}{b}$$

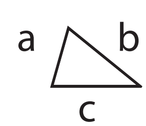
$$\frac{48}{21} = \frac{81}{b}$$

$$b \times 48 = 21 \times 81$$

$$48b = 1701$$

$$b = 35.4375$$





Next, I'll find the length of **c**.

$$\frac{A}{a} = \frac{C}{c}$$

$$\frac{48}{21} = \frac{68}{c}$$

$$c \times 48 = 21 \times 68$$
$$48c = 1428$$
$$c = 29.75$$

Remember to add the units of measure, in this case mm. The right answer is: **b = 35 mm and c = 30 mm**.

Example:

A picture measuring 3.5" high by 5" wide is to be enlarged so that the width is now 9 inches. How tall will the picture be?

In other words, the photo lab will be maintaining the aspect ratio; the rectangles representing the outer edges of the pictures will be similar figures. Set up the proportion and solve:

$$\frac{\text{height}}{\text{weight}} : \frac{3.5}{5} = \frac{h}{9}$$

$$9 \times 3.5 = 5 \times h$$
$$31.5 = 5h$$
$$6.3 = h$$

The picture will be 6.3 inches high.

In the first exercise above, the ratios were between corresponding sides, and the proportionality was formed from those pairs of sides; for instance,

(length of left-hand side on big triangle) :
(length of left-hand side on little triangle)
=
(length of base on big triangle) :
(length of base on little triangle

In the second exercise above, the ratios were between the two different dimensions, and the proportionality was formed from the sets of dimensions:

(original height) : (original length) =
(enlarged height) : (enlarged length)

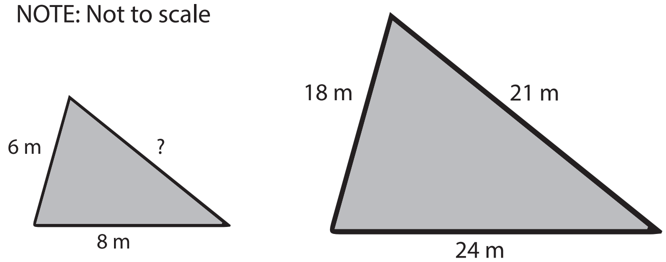
For many exercises, you will be able to set up your ratios and proportions in any of various ways. Just make sure that you label things well, clearly define your variables, and set things up in a sensible and consistent manner; this should help you dependably reach the correct solutions. If you're ever not sure of your solution, remember to plug it back into the original exercise, and verify that it works.

LESSON 6
Mathematical Reasoning

DIRECTIONS
Solve the following problems using proportions. The first one is set up for you. Write your answers on the answer sheet.

1. **When two triangles have the same shape but different sizes, they are said to be similar. With similar triangles, we can find the length of an unknown side by comparing known sides in a proportion.**

NOTE: Not to scale



$$\frac{\textit{short right side sm}\ \triangle}{\textit{short right side big}\ \triangle} = \frac{\textit{longer left side sm}\ \triangle}{\textit{longer left side big}\ \triangle}$$

$$\frac{6}{18} = \frac{?}{21}$$

What is the length of the unknown side?

2. **The scale of a map shows that 1 inch on the map equals 50 miles. If the distance between two points is 3 inches on the map, what is the actual distance between the points?**

3. **Elaine's car usually gets 27 miles per gallon of gas. How many gallons should she expect to use on a trip of 810 miles?**

Percents

Percents represent parts of things or relationships among things by comparing to 100. Percents are often used in business to describe changes in sales or to compare profit to income. As consumers, we frequently see percents used for discount sales. Use the understanding you already have from your experiences to estimate in problems that require percents.

In a percent problem, there is a number that represents the whole (or the base, which is the beginning amount), the percent (or the rate, which has a % sign), and the part (the number that occurs when the percent is applied to the whole). Normally, the whole is a larger number than the part, but the opposite can be true. In a problem statement, the word *of* usually precedes the whole.

There are several methods for solving percent problems. You might have learned a method that requires conversions (such as, changing a percent to a decimal). We cannot operate directly with a percent; it must be changed to a decimal or fraction. The ***proportion method*** of solving percent problems does the conversion for you. This method always uses the same proportion:

$$\frac{\textbf{Part}}{\textbf{Whole}} = \frac{\textbf{Percent}}{\textbf{100}}$$

Think of the above ***ratio-proportion set up as a formula***. Write it down before you begin the problem and then plug in your numbers where they go.

LESSON 6
Mathematical Reasoning

To solve a percent problem:

1. Identify which of the part, whole, and percent are available in the problem statement.
2. Set up the proportion.
3. Solve the proportion. Multiply the two numbers on the diagonal, then divide by the other number. For example, find 6% of 325. 6% is the percent and 325 is the whole.

Example: We'll find the part.

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$$

$$\frac{\text{part}}{325} = \frac{6}{100}$$

$$\frac{325 \times 6}{100} = \text{Part}$$

$$195 = \text{Part}$$

The answer: 195 is 6% of 325

ASSIGNMENT 4

DIRECTIONS

Solve the following percent problems using the examples already given. Show your work on a separate sheet of paper. Circle your answer, then make sure your answer is reasonable. Write your final answers on the lines provided. When you find the answer indicate whether you found the *part*, *whole* or *percent*.

1. **What is 50% of 220?** ____________________
2. **What is 17.5% of 36?** ____________________
3. **What is 110% of $54,000?** ________________
4. **What percent of 300 is 60?** ______________
5. **What percent of 50 is 75?** ________________
6. **78.3 is what percent of 120?** _____________
7. **42 is 75% of what number?** ______________
8. **168 is 12% of what number?** ______________
9. **1715.8 is 115% of what number?** __________

LESSON 6
Mathematical Reasoning

Percent of Change

Sometimes, a ***percent of change*** is called for. To solve such a problem, first find the difference between the beginning and the ending numbers (subtract). Using the difference as the part and the beginning number (original) as the whole, use the proportion method to find the percent change. Remember to use the set-up formula for ratio and proportion.

Example: What is the percent of increase from $1000 to $1250?

- **Find the difference between the beginning and ending number. This is the amount of increase.**

$1250 – $1000 = $250

$$\frac{\text{amount of increase}}{\text{original}} = \frac{\$250 \text{ (part)}}{\$1000 \text{ (whole)}}$$

$$\frac{\text{percent}}{100} = \frac{250 \times 100}{1000} = 25\%$$

The answer is 25% of an increase.

ASSIGNMENT 5

DIRECTIONS

Solve these problems requiring percent change, and write your answer on the line.

1. **Karen weighed 178 pounds on January 1. On April 1, she weighed 165 pounds. By what percent did her weight decrease?**

   ____

2. **Lift-A-Lot Gym raised membership rates from $30 to $35 per month. What is the percentage increase in their rate?**

   ____

3. **Lounge-A-Lot Recliner Company marked down the previous year's models from $500 to $425. By what percent did they mark down the chairs?** ____________________

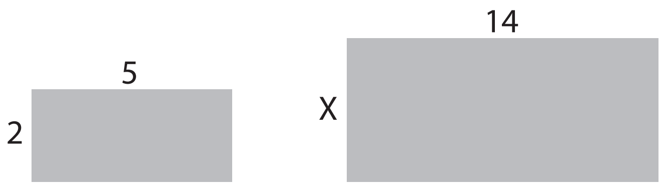
4. **The Klammon family bought their home for $92,000. It increased in value by 6% while they lived there. What was the home's value when they sold it?**

   ____

LESSON 6
Mathematical Reasoning

ASSIGNMENT 6

1. **Two rectangles are similar and the dimensions shown are in centimeters.**



What is the measure of x, in centimeters?

A. 4.0
B. 5.6
C. 8.4
D. 11.0

2. **Dominic earns $285 per week plus and 8% commission rate on all his sales. If Dominic sells $4213 worth of merchandise in one week, how much will his total earning for the week be?**

A. $337.04
B. $359.84
C. $513.00
D. $622.0

3. **The probability of rain on each of the next three days is given in the table.**

Probability of Rain Each Day

Day	Tuesday	Wednesday	Thursday
Probability of rain	30%	45%	50%

What is the percent probability that it will rain all three days? ____________________

4. **You shot the basketball 25 times and made 68% of your shots. How many shots did you make and how many did you miss?**

A. Made 7 shots: missed 18 shots
B. Made 18 shots: missed 7 shots
C. Made 17 shots: missed 8 shots
D. Made 8 shots: missed 17 shots

★

EVERY ACCOMPLISHMENT STARTS WITH THE DECISION TO TRY.

★

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www.gedtestingservices.com