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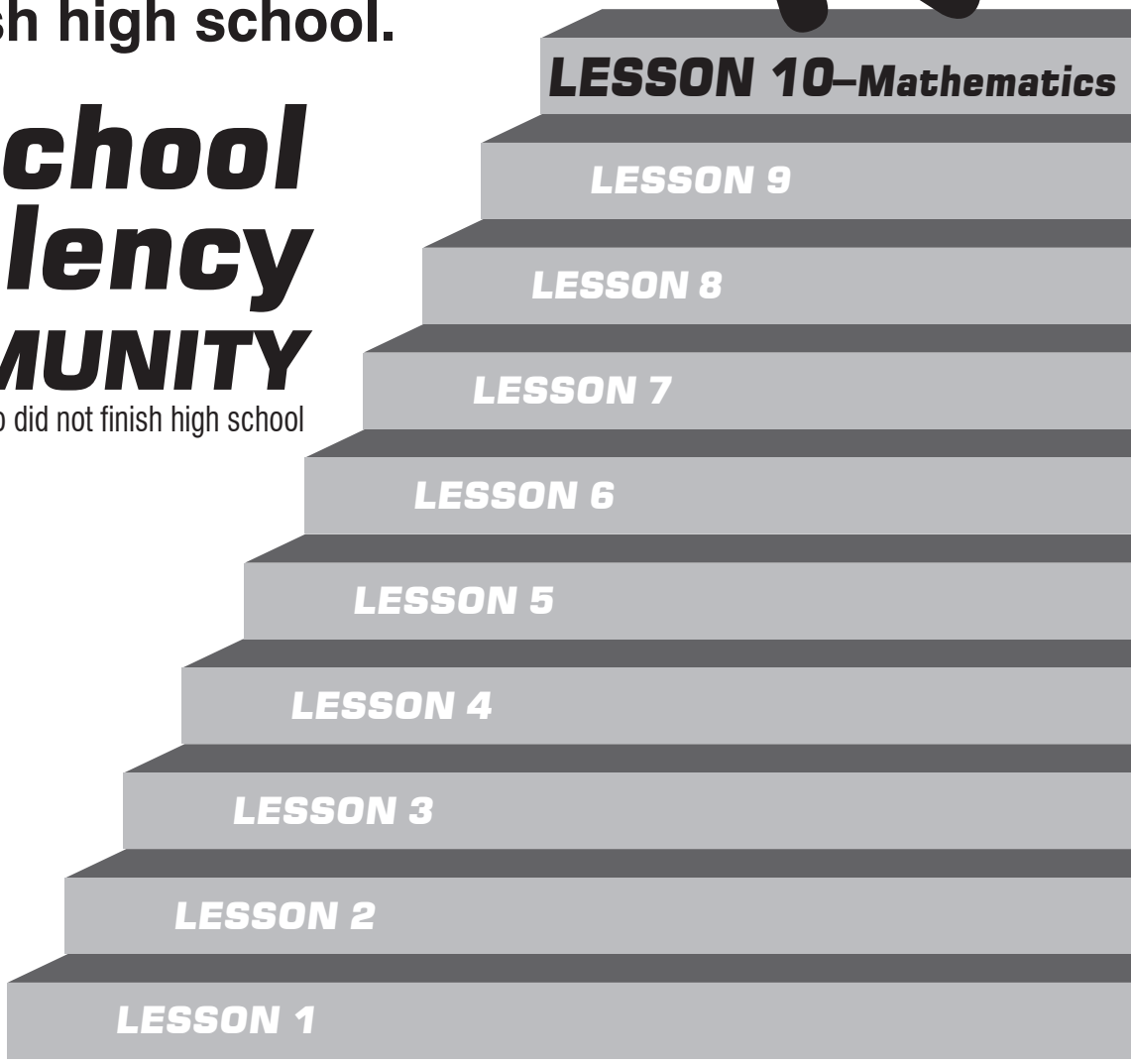


Tremendous Journey– YOU MADE IT!

LESSON 10
Mathematical Reasoning

ASSIGNMENT 1

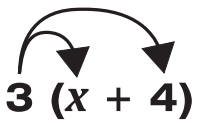
The Distributive Property

This topic is really part of studying the ***Order of Operations (Lesson 2)***, but simplifying with parentheses is probably the sub-topic that causes students the most difficulty, so this lesson is meant to provide a little extra help in this area.

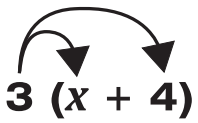
When simplifying expressions with parentheses, you will be applying the ***Distributive Property***. That is, you will be distributing over (multiplying through) a set of parentheses in order to simplify a given expression. I will walk you through some examples of increasing difficulty, and you should note, as this lesson progresses, the importance of simplifying as you go and of doing each step neatly, completely, and exactly.

Simplify 3(*x* + 4)

To "simplify" this, I have to get rid of the parentheses. The Distributive Property says to multiply the 3 onto everything inside the parentheses. I



sometimes draw arrows to emphasize this:



Then I multiply the 3 onto the ***x*** and onto the 4:

3(x) + 3(4)

3x + 12

Written all in one line, this would look like:

3 (*x* + 4) = 3(*x*) + 3(4) = 3*x* + 12

The most common error at this stage is to take the 3 through the parentheses but only onto the ***x***, forgetting to carry it through onto the 4 as well. If you need to draw little arrows to help you remember to carry the multiplier through onto ***every***thing inside the parentheses, then use them!

Simplify –2(x – 4)

I have to take the –2 through the parentheses. This gives me:

–2(x – 4)
–2(x) – 2(–4)
–2x + 8

The common mistake with this type of problem is to lose a "minus" sign somewhere, such as doing "–2(x – 4) = –2(x) – 2(4) = –2x – 8". Did you notice how the "–4" somehow turned into a "4" when the –2 went through the parentheses? That's why the answer ended up being wrong.

Be careful with the "minus" signs! Until you are confident in your skills, take the time to write out the distribution, complete with the signs, as I did.

–2(x – 4)
–2(x) – 2(-4)
–2x + 8

If you have difficulty with the subtraction, try converting it to addition of a negative:

–2(x – 4)
–2(x + [-4])
–2(x) + (-2)(-4)
–2x + 8

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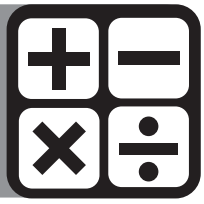

Do as many steps as you need to, in order to consistently get the correct answer.

Simplify –(x – 3)

I have to take the "minus" through the parentheses. Many students find it helpful to write in the little understood "1" before the parentheses:

–1(x – 3)

I need to take a **–1** through the parentheses:

–(x – 3)
–1(x – 3)
–1(x) – 1(–3)
–1x + 3
–x + 3

Note that "–1x + 3" and "–x + 3" are technically the same thing. However, when taking test the "1" will not be placed before the 'x' in answers.

Simplify 2 + 4(x – 1)

The order of operations tells me that multiplication comes before addition. I can't do the "2 +" until I have taken the 4 through the parentheses.

2 + 4(x – 1)
2 + 4(x) + 4(–1)
2 + 4x + (–4)
2 – 4 + 4x
–2 + 4x
4x – 2

"4x – 2" and "–2 + 4x" are the same answer; however, most tests expect the answer to be written in "descending order" (with the variable term first, and then the plain number).

Parentheses inside of parentheses are called "nested" parentheses. The process of simplification works the same way as in the simpler examples on the previous page, but you do need to be a little more careful as you work your way through the grouping symbols.

Simplify 4[x + 3(2x + 1)]

With nested parentheses like this, the safest plan is to work from the inside out. So I'll take the 3 through the inner parentheses first, before I even think about dealing with the 4 and the square brackets. I'll also simplify as much as I can as I go along. Note that I write each step out completely as I go:

4[x + 3(2x + 1)]
4[x + 3(2x) + 3(1)]
4[x + 6x + 3]
4[7x + 3]
4[7x] + 4[3]
28x + 12

Simplify 9 – 3[x – (3x + 2)] + 4

I won't do anything with the "9 –" or the "+ 4" until I simplify inside the brackets and parentheses. I'll work from the inside out:

9 – 3[x – (3x + 2)] + 4
9 – 3[x – 1(3x + 2)] + 4
9 – 3[x – 1(3x) – 1(2)] + 4
9 – 3[x – 3x – 2] + 4
9 – 3[–2x – 2] + 4
9 – 3[–2x] – 3[–2] + 4
9 + 6x + 6 + 4
6x + 19

It is not required that you write out this many (or this few) steps. You should be careful to do one step at a time, though, writing things out completely and simplifying as you go. You should do as many steps as you need in order to consistently arrive at the correct answer.

Let me stress once again to "simplify" problems successfully, you need to:

- Take the time to write out each step.
- Work from the inside out, and be careful with the "minus" signs.
- Don't forget the ***Order of Operations***,
- And don't make the mistake of confusing "simplifying" with "***solving***".

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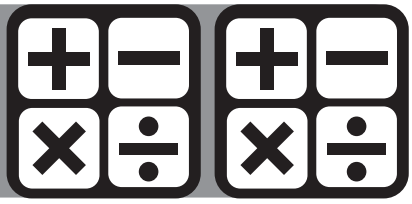
Mathematical Reasoning



ASSIGNMENT 2

DIRECTIONS

Simplify the expressions. Write your answer in the space provided. Show your work on a separate sheet of paper.

1. $3-3(x-2)$ _____
2. $-(1-5n)-7$ _____
3. $8+7(7n-4)$ _____
4. $4+5(3x-3)$ _____
5. $5-2(8x+4)$ _____
6. $-5(-6-7n)$ _____
7. $7(5n-1)-2$ _____
8. $-7(8x-3)$ _____
9. $-(4+6x)-3$ _____
10. $5(-8n+5)-4$ _____
11. $3-6(1+n)$ _____
12. $5+2\{[3+(2x-1)+x]-2\}$ _____

Radical Expressions

Vocabulary to know

Radical Expression–An expression containing a square root

Radicand–A number or expression inside the radical symbol

Factors–Factors are the numbers you multiply together to get another number.

Example: **What are the factors of 12?**

1 x 12 = 12; 2 x 6 = 12; 3 x 4 = 12

The factors of 12 are 1,2,3,4,6,12

Factoring—to factor—to expand–This is the process of finding factors of a number or an expression

To simplify a square root: make the number inside the square root as small as possible (but still a whole number)

Prime Number–A Prime Number can be divided evenly only by 1 or itself. And it must be a whole number greater than 1.

Example: **1, 3,7,13 are prime numbers.**

Composite Number–A number that can be divided evenly by numbers other than 1 or itself.

Example: **4, 8, 16, 25 are composite numbers.**

Square Roots–Radicals

"Roots" (or "radicals") are the "opposite" operation of applying exponents.

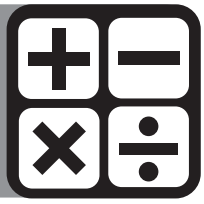
For instance, if you square 2, you get 4, and if you "take the square root of 4", you get 2; if you square 3, you get 9, and if you "take the square root of 9", you get 3:

$$2^2 = 4, \text{ so } \sqrt{4} = 2$$

$$3^2 = 9, \text{ so } \sqrt{9} = 3$$

The "√" symbol is called the "radical"symbol. The expression "√9" is read as "root nine," "radical nine," or "the square root of nine."

LESSON 10
Mathematical Reasoning



You can raise numbers to powers other than just 2; you can cube things, raise them to the fourth power, raise them to the 100th power, and so forth. In the same way, you can take the cube root of a number, the fourth root, the 100th root, and so forth. To indicate some root other than a square root, you use the same radical symbol, but you insert a number into the radical, tucking it into the "check mark" part. For instance:

$$4^3 = 64, \text{ so } \sqrt[3]{64} = 4$$

The "3" in the above is the "index" of the radical; the "64" is "the argument of the radical", also called "the radicand." Since most radicals you see are square roots, the index is not included on square roots. While "$\sqrt[2]{}$" would be technically correct, it is never used.

You can take any counting number, square it, and end up with a nice neat number. But the process doesn't always work going backwards. For instance, consider $\sqrt{3}$, the square root of three. There is no nice neat number that squares to 3, so $\sqrt{3}$ cannot be simplified as a nice whole number.

If you are using this in a word problem to find the rate of speed, then you would use your calculator and find the approximation of $\sqrt{3}$. The square root of 3 is about 1.732050808. Then you would round the appropriate decimal place—say tenths—and use a real-world unit or label, like "1.7 minutes/seconds." If you are asked for and exact value the answer is $\sqrt{3}$.

Simplifying Square-Root Terms

To simplify a square root, you "take out" anything that is a "perfect square"; that is, you take out front anything that has two copies of the same factor:

$$\sqrt{4} = \sqrt{2^2} = \sqrt{2 \times 2} = 2$$

$$\sqrt{49} = \sqrt{7^2} = \sqrt{7 \times 7} = 7$$

Note that the value of the simplified radical is *positive*. While either of +2 and –2 might have been squared to get 4, "the square root of four" is *defined* to be only the positive option, +2. When you solve the equation $x^2 = 4$, you are trying to find *all* possible values that might have been squared to get 4. But when you are just simplifying the expression $\sqrt{4}$, the ONLY answer is "2".

Squares	Square Roots	Cubes
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$
$7^2 = 49$	$\sqrt{49} = 7$	$7^3 = 343$
$8^2 = 64$	$\sqrt{64} = 8$	$8^3 = 512$
$9^2 = 81$	$\sqrt{81} = 9$	$9^3 = 729$
$10^2 = 100$	$\sqrt{100} = 10$	$10^3 = 1000$
$11^2 = 121$	$\sqrt{121} = 11$	$11^3 = 1331$
$12^2 = 144$	$\sqrt{144} = 12$	$12^3 = 1728$

ASSIGNMENT 3

DIRECTIONS

Simplify the *radical* or find the *square root*.

1. $\sqrt{289} =$ _____
2. $\sqrt{36} =$ _____
3. $\sqrt{225} =$ _____
4. $\sqrt{169} =$ _____
5. $\sqrt{196} =$ _____
6. $\sqrt{256} =$ _____
7. $\sqrt{16} =$ _____
8. $\sqrt{144} =$ _____

Factoring a Radical

Sometimes the radicand of a radical is not a perfect square, but it may "contain" a square amongst its ***factors***. To simplify, you need to ***factor*** the argument and "take out" anything that is a square; you find anything you've got a pair of inside the radical, and you move it out front.

Example 1: **Simplify $\sqrt{72}$**

Since 72 factors as 2×36 and since 36 is a perfect square, then:

$$\sqrt{72} = \sqrt{(2 \times 36)} = \sqrt{[2 \times (6\times6)]} = 6\sqrt{2}$$

Since there had been only one copy of the factor 2 in the factorization $2\times6\times6$, the left-over 2 couldn't come out of the radical and had to be left behind.

Example 2: **Simplify $\sqrt{75}$**

$$\sqrt{75} = \sqrt{25} \times 3 = 5\sqrt{3}$$

You don't have to factor the radicand all the way down to prime numbers when simplifying. As soon as you see a pair of factors or a perfect square, you've gone far enough.

Variables in a radical's argument are simplified in the same way: whatever you've got a pair of can be taken "out front."

Example 3: **Simplify $\sqrt{16x^4}$**

$$\sqrt{16x^4} = \sqrt{4 \cdot 4 \cdot xx \cdot xx} = 4xx = 4x^2$$

LESSON 10
Mathematical Reasoning

ASSIGNMENT 4

DIRECTIONS
Simplify the radical. Show your work.

1. $\sqrt{28}$ _____
2. $\sqrt{12}$ _____
3. $\sqrt{20}$ _____
4. $\sqrt{9x}$ _____

Multiplying Square Roots

Multiply the radicals and then simplify by writing with no more than one radical.

Example 4: **Simplify by writing with no more than one radical: $\sqrt{2} \cdot \sqrt{8}$ *or* $\sqrt{2}\sqrt{8}$**

$$\sqrt{2}\sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{16} = \sqrt{4 \cdot 4} = 4$$

Example 5: **Simplify by writing with no more than one radical: $\sqrt{6}\sqrt{15}\sqrt{10}$**

$$\begin{aligned}\sqrt{6}\sqrt{15}\sqrt{10} &= \sqrt{6 \cdot 15 \cdot 10} \\ &= \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 2} \\ &= \sqrt{(2 \cdot 2) \cdot (3 \cdot 3) \cdot (5 \cdot 5)} \\ &= 2 \cdot 3 \cdot 5 \\ &= 30\end{aligned}$$

ASSIGNMENT 5

DIRECTIONS
Simplify.

1. $\sqrt{3} \cdot \sqrt{11} =$ _____
2. $\sqrt{5} \cdot \sqrt{5} =$ _____
3. $\sqrt{3} \cdot \sqrt{18} =$ _____
4. $\sqrt{15} \cdot \sqrt{6} =$ _____
5. $\sqrt{18} \cdot \sqrt{14x} =$ _____
6. $\sqrt{3x} \cdot \sqrt{12} =$ _____

7. **Cilcia are very thin, hair-like projections from cells. They are 2.0×10^{-4} millimeters wide. What is the maximum number of cilia that would fit side by side—without overlapping—across a microscope slide that is 25 millimeters wide? Choose the correct answer in scientific notation.**
   A. 8.0×10^{-6}
   B. 1.25×10^{-3}
   C. 8.0×10^{2}
   D. 1.25×10^{5}

LESSON 10
Mathematical Reasoning

Adding and Subtracting Square Roots

Just as with "regular" numbers, square roots can be added together. But you might not be able to simplify the addition all the way down to one number. Just as "you can't add apples and oranges," so also you cannot combine "unlike" radicals. To add radical terms together, they have to have the same radical part. **$2\sqrt{3} + 3\sqrt{3}$**

Example 1: **Simplify $2\sqrt{3} + 3\sqrt{3}$**

Since the radical is the same in each term, you can combine the terms. Keep the radicond and add or subtract the coefficients.

$$2\sqrt{3} + 3\sqrt{3} = (2+3)\sqrt{3} = 5\sqrt{3}$$

Example 2: **Simplify $\sqrt{3} + 4\sqrt{3}$**

The radical part is the same in each term. Keep the radical and add the coefficients. The first term means "one square root of three," I'll insert the ***"understood" "1"***:

$$\sqrt{3} + 4\sqrt{3} = 1\sqrt{3} + 4\sqrt{3} = (1+4)\sqrt{3} = 5\sqrt{3}$$

Don't assume that expressions with unlike radicals cannot be simplified. It is possible that, *after simplifying the radicals*, the expression can indeed be simplified.

Example 3: **Simplify: $\sqrt{9} + \sqrt{25}$**

Simplify each radical expression and then add the coefficients. The radicands 9 and 25 are perfect squares.

$$\sqrt{9} + \sqrt{25} = 3 + 5 = 8$$

ASSIGNMENT 6

DIRECTIONS

Simplify. Show your work on a separate sheet of paper.

1. $3\sqrt{2} + 9\sqrt{2} =$ _____
2. $8\sqrt{5} - 3\sqrt{5} =$ _____
3. $2\sqrt{10} - 7\sqrt{40} =$ _____
4. $\sqrt{24} + \sqrt{54} =$ _____
5. $7\sqrt{3} + 9\sqrt{3} =$ _____
6. $7\sqrt{5} - 3\sqrt{5} =$ _____

LESSON 10
Sample Test Questions–Mathematical Reasoning

SAMPLE TEST QUESTIONS

Questions 1 and 2 refer to the following information.

After several interviews, Dedra has narrowed her choices to two jobs. She has used the table below to organize data that will help her make the final decision.

	STARTING SALARY	RAISES	TRANSPORTATION COSTS
Applied Systems	$26,000 per year	4% after six months; 6% of current salary after one year	20 gallons of gas per month plus $50 per month for parking
NetWorks	$2200 per month	10% after one year	Bus fare of $2 per day plus subway fare of $4.50 per day

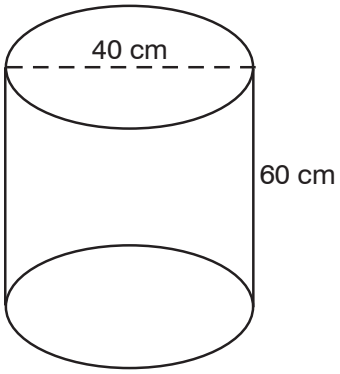
1. If Dedra chooses the job at NetWorks, what would be her ANNUAL salary after she had worked for them for one year?

A. $2,420
B. $22,000
C. $24,200
D. $26,400
E. $29,040

2. What can Dedra expect her monthly cost for transportation to be if she chooses to work for Applied Systems?

A. $ 50
B. $ 70
C. $ 80
D. $170
E. Not enough information is given.

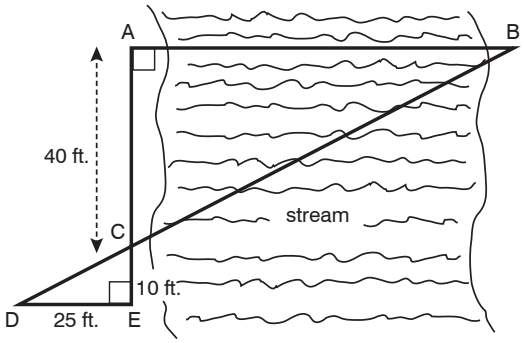
3. A painter mixes gallons of paint in a large cylindrical bucket so that there will be no difference in color between individual gallons. The bucket is shown below.



If one gallon of paint has a volume of approximately 8000 cm3, what is the maximum number of whole gallons of paint that can be poured into the bucket?

A. 3
B. 7
C. 9
D. 11
E. 37

4. A surveyor made the measurements shown in the diagram below.

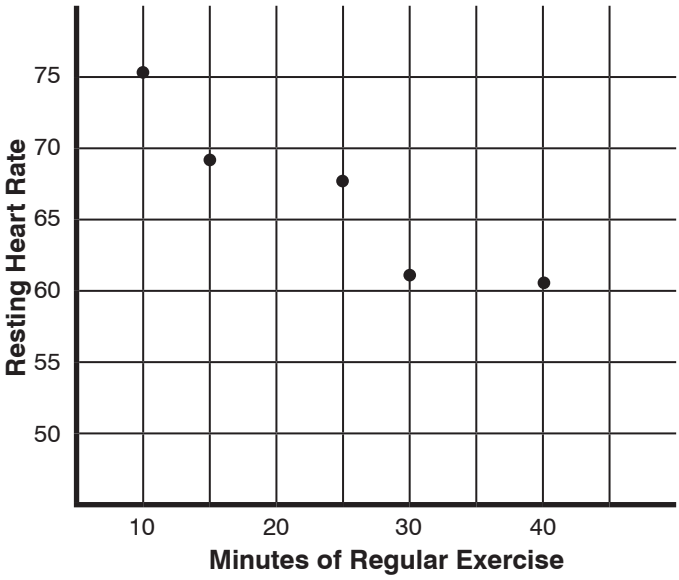


In feet, what is the measure of AB, the straight-line distance across the stream?

A. 50
B. 75
C. 80
D. 100
E. 150

LESSON 10
Sample Test Questions–Mathematical Reasoning

Questions 5 and 6 refer to the following information and graph.



Five people aged 40 to 50 who exercised regularly had their resting heart rates recorded. The results are shown in the graph above.

5. **Based on the graph above, which of the following would be a reasonable resting heart rate for a person aged 40 to 50 who regularly exercised for 20 minutes?**
   A. 77
   B. 67
   C. 57
   D. 52
   E. 48

6. **Which of the following statements is a reasonable conclusion based on the data in the graph above?**
   A. The resting heart rate of a person in this age group is not affected by regular exercise.
   B. Regular exercise is likely to increase the resting heart rate of a person in this age group.
   C. A person's resting heart rate is related to the person's age.
   D. Regular exercise by a person in this age group would likely lower her or his resting heart rate.
   E. As a person gets older, her/his resting heart rate will increase.

7. **Cabezos Construction Company has a clause in the contract for its latest building that rewards the company for completing the project early. The company will receive a bonus of $1000 if they are one day early and an additional $1500 for each day more than one. The formula is $B = 1000 + 1500(d - 1)$, where B is the amount of the bonus and d is the number of days they complete construction early. If the company completes the project 10 days early, what will be the amount of their bonus?**
   A. $15,000
   B. $14,500
   C. $14,000
   D. $13,500
   E. $10,000

LESSON 10
Sample Test Questions–Mathematical Reasoning

8. Shane is working with a spreadsheet on his computer. The spreadsheet will calculate the cost of the wood trim around rectangular windows based on the dimensions of the window and the price of the wood. The following entries have been made.

Length of window in feet	Width of window in feet	Price per foot of wood trim	Cost of trim for window
A7	B7	C7	

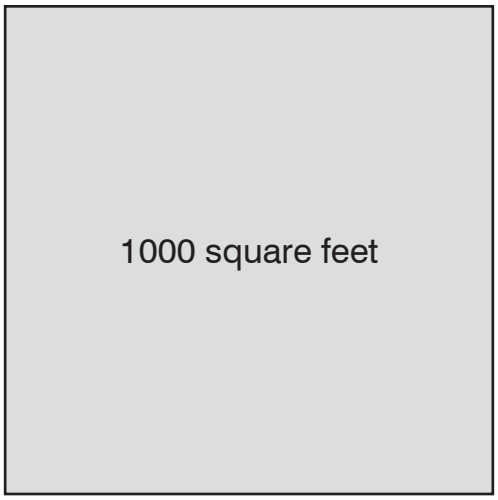
Shane wants to enter a formula in the last column so that the spreadsheet will calculate the final cost of the job. Which of the following formulas should he enter? Helpful information can be found on the Formula page at the beginning of this group of questions.

A. A7 x B7 x C7
B. (2 x A7 + 2 x B7) x C7
C. A7 + B7 + C7
D. (A7 + B7) x C7
E. A7 x B7 + C7

The following three questions are specific examples of items likely to be found on the calculator portion of the new Mathematics Test.

9. Last month the balance in Tisha's checkbook was $1219.17. Since then she has deposited her latest paycheck of $2425.66 and written checks for $850.00 (rent), $235.89 (car payment), and $418.37 (credit card payment). What is the current balance in Tisha's checking account?

A. $ 921.40
B. $2140.57
C. $3215.27
D. $3929.92
E. $5149.09



10. A farmer wants to plant a rectangular field that will cover an area of 1000 square feet. To minimize the amount of fencing around the field, she knows the rectangle should be a square. To find the length of a side of a square (*s*) when its area (*A*) is know, the following formula is used: $s = A$. Rounded to the nearest tenth of a foot, what would be the length of a side of the farmer's square field if it contains an area of 1000 square feet?

A. 28.7
B. 29.4
C. 31.6
D. 36.2
E. 41.8

11. The warehouse is shipping 6832 calculators. If each box can contain 28 calculators, how many boxes will be needed for the shipment?

A. 79
B. 154
C. 169
D. 244
E. 239

LESSON 10
Sample Test Questions–Mathematical Reasoning

The following four questions are specific examples of alternate format items.

12. Byron purchased a $5000 certificate of deposit (CD) at his local bank. The CD will pay him 7% simple interest at the end of two years. In dollars, how much *INTEREST* will Byron have earned from his CD at the end of the two year period?

Mark your answer in the circles in the grid on your answer sheet.

13. To test the effect of a fertilizer, pea plants are measured each week. If one pea plant measured 28.4 centimeters this week and 16.1 centimeters last week, by how many centimeters did it grow during the last week?

Mark your answer in the circles in the grid on your answer sheet.

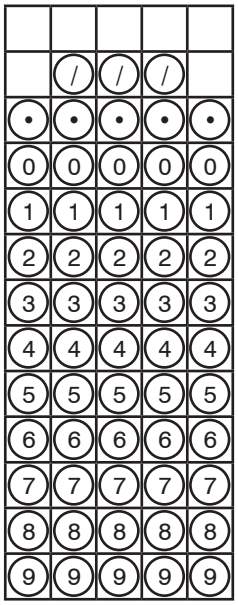
14. One ingredient in the sauce Kyle is preparing for tonight's dinner is ½ teaspoon of red pepper. If the recipe he is using is designed to make enough sauce for 8 servings but Kyle is only making 4 servings, what fraction of a teaspoon of red pepper should he use?

Mark your answer in the circles in the grid on your answer sheet.

15. Show the location of the point whose coordinates are (3, -4).

Mark your answer on the coordinate plane grid on your answer sheet.

12.



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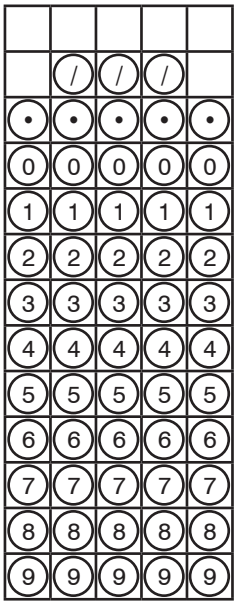

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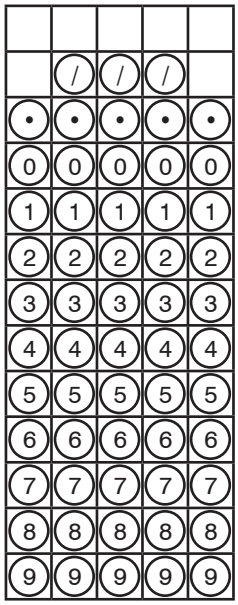
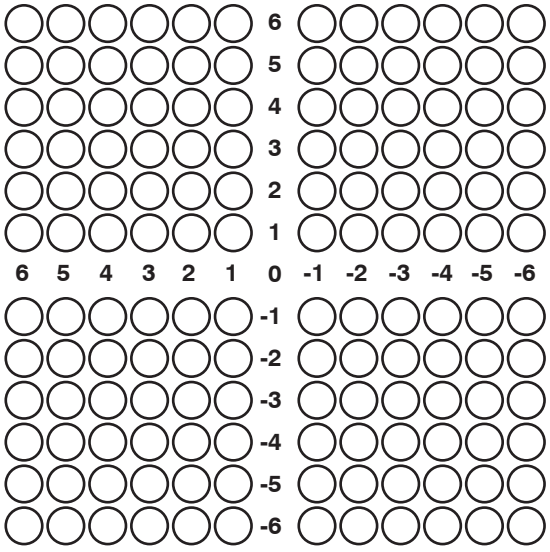


15.



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